# Modélisation non-locale de la résistance en compression des composites architecturés

## *Non-Local modelling of the compressive strength of composite structures*

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### Résumé

Cet article propose un modèle numérique homogénéisé d'éléments finis non locaux, similaire à la théorie du gradient II de Mindlin [1], pour évaluer la résistance à la compression des composites fibres longues carbone / époxy à l'échelle mésoscopique / structurelle. Le cadre de cette modélisation non locale est plus général que celui de [2] pour simuler le phénomène de microbuckling dans les composites UD et tissés. Le modèle numérique non local développé est implémenté dans le sous-programme User Elément (UEL) d'ABAQUS ©, ce qui permet de simuler le comportement de cas 2D et 3D. Un 2D élément non locale (NL U32) super-paramétrique continu ($C^1$) est développé pour le cas élastique isotrope linéaire. Différents résultats de tests sont présentés pour valider le modèle FE non local pour les cas 2D, en comparant les résultats d'un élément non local homogénéisé avec un élément iso-paramétrique d'Abaqus et en comparant le comportement mécanique avec une microstructure composite 2D discrétisée à l'aide d'éléments classiques d'ABAQUS.

### Abstract

A homogenized non-local finite element model is proposed in this article, similar to Mindlin's II gradient theory [1] to assess the compressive strength of the carbon/epoxy long fiber composite at the mesoscopic/structural scale. The framework of this non-local modelling is more general that of [2] to assess microbuckling phenomenon in UD and woven composites. The developed nonlocal numerical model is implemented in User Element (UEL) subroutine for analysis in ABAQUS ©, which permits to simulate the behavior of 2D and 3D cases. A 2D continuous ($C^1$) super-parametric non-local element (NL U32) is developed for linear isotropic elastic case. Various tests results are presented to validate the non-local FE model for 2D cases, by comparing results of homogenized non-local element with ABAQUS © iso-paramteric element and then by comparing mechanical behavior/response with 2D composite microstructure, modelled using ABAQUS © classical elements.

**Mots Clés :** Modèle non local, compression, composites à fibres longues
**Keywords :** Non-local model, compression, Long-fiber composites

## 1. Introduction

The compressive failure of long carbon fiber composites is due to complex mechanisms. The knowledge of the phenomena is important for the design of composite structures [3], because during the design of composite parts, compressive strength and stiffness of laminates is assumed less than their tensile strength, which is not justified. Competing modes of compressive failure exist, including *delamination, fiber failure and elastic-plastic microbuckling*. '*Elastic microbuckling'* is a local instability where the matrix deforms in simple shear, whereas '*Plastic microbuckling'* is a shear buckling instability, which occurs at sufficiently large strains for the matrix to deform in non-linear manner [4]. In general, the main parameters that influence the microbuckling and kink/shear band formation are: a) Matrix physical non-linearity and b) Presence of fiber initial wavy imperfection/undulation [16-18].

There are many articles in the literature regarding the modeling of composites compressive behavior, particularly the microbuckling phenomenon as a local instability of UD composites (local models: [5-7]). Only few researchers modeled the mechanism at the structural / mesoscopic scale (Non-local models: [8-10]). For example, Drapier et al. [2], proposed a 2D homogenized model (Eq. 1), which takes into account fiber initial alignment defects, matrix plasticity and structural parameters.

$$-\int_{\Omega}\{f E_f\, r_{gf}^{\mathbf{2}} v''\, \delta v'' + \boldsymbol{S}\cdot \delta\boldsymbol{E}\} d\Omega + \langle \boldsymbol{F}\cdot\delta\boldsymbol{u}\rangle = 0 \qquad \forall \delta\boldsymbol{u} \qquad \text{(Eq. 1)}$$

where, $f$ is fiber volume fraction, $E_f$ is fiber Yong's modulus, $r_{gf}=\sqrt{\frac{I}{S_f}}$ is fiber gyration radius, $v''$ is fiber curvature field, $\boldsymbol{S}$ is Second Piola Kirchhoff stress tensor, $\boldsymbol{E}$ is the Green Lagrange strain tensor. The model is successful in predicting the *elastic microbuckling modes*, but the model is 2D and assumes that, the microbuckling is periodic in fiber direction, just one gradient in thickness direction is taken into account. Consequently, not possible to compare test results obtained with the complex real 3D structures. Moreover, the prediction of both the 'distribution' and 'amplitudes' of fiber initial imperfection is still not well known [11]. Hence, it is necessary to extend the model of Drapier et al. [2]. The numerical tool should be able to apprehend instability in complex three-dimensional gradient situations such as near a hole edge and should take into account the distribution of defects at the structural scale. The other difficulty is in extending the theory to woven composites, where the wavelength of the microstructure interacts with instability. In addition, under complex loads, damage contributes to the modification of stiffness and non-linearities and influences both modes and critical loads.

Therefore, a new homogenized non-local finite element model is proposed, more general of [2] to simulate compressive behavior of composite materials at macro/mesoscopic scale, which permits to take into account the microstructural effects. The developed non-local numerical model is implemented in User Element (UEL) subroutine for analysis in ABAQUS ©, which permits to simulate the behavior of 2D and 3D cases. At present, a 2D continuous ($C^1$) super-parametric non-local element (NL U32) is developed for linear isotropic elastic case (both the matrix and fiber is assumed to be isotropic and elastic) and validated with the ABAQUS © elements. Addition of non-linearities (material and geometrical) is in progress and the validation will be done as soon as possible, and then extension to the development of 3D non-local element.

The theoretical and numerical parts of the model are presented in the section 2. Validation of the developed finite element for 2D case is performed by comparing results of homogenized non-local super-parametric element with one of the ABAQUS © iso-paramteric elements and then by comparing mechanical behavior/response with 2D composite microstructure, modelled using ABAQUS © elements as a final validation (section 3).

## 2. Homogenized non-local numerical model

### 2.1 Theoretical part

The energy of local bending corresponds to the first term in (Eq. 1). This term has been obtained by Gardin et al. [17] with an asymptotic development. But, it is very restrictive, and corresponds to just UD ply under compression. Therefore, it is necessary to extend the model for more general realistic case, taking into account the fiber curvature field in more complex mesostructure, for example: 2D and 3D woven, where fiber curvature field is not only restricted in one direction, but are randomly oriented in multiple directions, during compression or torsion-compression, as shown in (Fig . 1):

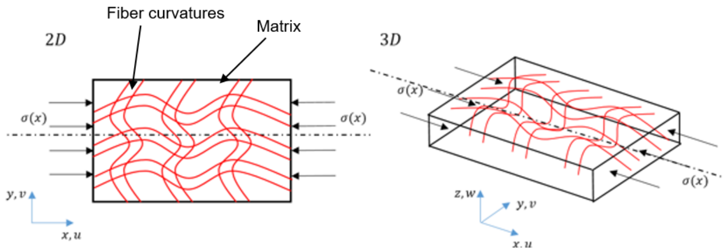



*Fig. 1. 2D and 3D woven composite with fibers curvatures in multiple directions loaded under compression*

In the Mindlin's second order strain gradient theory [1], both the curvature and strain generate an energy of deformation. Hence, this theory is used as a main reference to develop our homogenized nonlocal model. The kinematics is defined by the displacement field (classical), $\boldsymbol{u} = u(x, y, z)\boldsymbol{e}_1 + v(x, y, z)\boldsymbol{e}_2 + w(x, y, z)\boldsymbol{e}_3$ and the generalized fiber curvature field is defined by, $\boldsymbol{\kappa}$ :

$$\boldsymbol{\kappa} = \left[\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 u}{\partial x \partial y}, \frac{\partial^2 v}{\partial x^2}, \frac{\partial^2 v}{\partial y^2}, \frac{\partial^2 v}{\partial x \partial y}\right]_{2D} \qquad \text{(Eq. 2)}$$

$$\boldsymbol{\kappa} = \left[\frac{\partial^2 u}{\partial x^2}, \frac{\partial^2 u}{\partial y^2}, \frac{\partial^2 u}{\partial z^2}, \frac{\partial^2 u}{\partial x \partial y}, \frac{\partial^2 u}{\partial x \partial z}, \frac{\partial^2 u}{\partial y \partial z}, \frac{\partial^2 v}{\partial x^2}, \frac{\partial^2 v}{\partial y^2}, \frac{\partial^2 v}{\partial z^2}, \frac{\partial^2 v}{\partial x \partial y}, \frac{\partial^2 v}{\partial x \partial z}, \frac{\partial^2 v}{\partial y \partial z}, \frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2}, \frac{\partial^2 w}{\partial z^2}, \frac{\partial^2 w}{\partial x \partial y}, \frac{\partial^2 w}{\partial x \partial z}, \frac{\partial^2 w}{\partial y \partial z}\right]_{3D} \qquad \text{(Eq. 3)}$$

The new variables, $\boldsymbol{\kappa} = \kappa_{ijk}$ , *the fiber curvature field or higher order bending strains* in multiple directions and $\overline{\overline{\boldsymbol{C}_f}}$ , *the local fiber bending stiffness matrix* are introduced which depends on the complex mesostructure. Therefore, using principle of virtual work (PVW), a new variational formulation of homogenized non-local numerical model in order to assess microbuckling problem at mesoscopic scale is written as (Eq. 4):

$$-\int_{\Omega\rightarrow 3D}\{\boldsymbol{\zeta}\cdot\delta\boldsymbol{\kappa} + \boldsymbol{S} : \delta\boldsymbol{E}\}d\Omega + \langle \boldsymbol{F}\cdot(\delta\boldsymbol{u}\,,\delta\boldsymbol{\kappa})\,\rangle = 0 \qquad \forall\delta\boldsymbol{u}\,,\forall\delta\boldsymbol{\kappa} \qquad \text{(Eq. 4)}$$

Where, $\boldsymbol{\zeta}$ is distributed bending moment (DBM) due to the fiber curvature fields, which leads to bending of fiber. It is related to the local fiber bending stiffness matrix by a linear law (Eq. 5) as a first approximation, where, $\overline{\overline{\boldsymbol{C}_f}}$ contains a non-local parameters of the mesostructure material. Tensors: $\boldsymbol{S}$ is the Second Piola Kirchhoff stress tensor, $\boldsymbol{E}$ is the Green Lagrange strain tensor, and $\boldsymbol{F}$ is the generalized external load vector. The 1st term in (Eq. 4) corresponds to '*internal fiber bending energy*', 2nd term corresponds to '*internal in-plane/classical strain energy*' and 3rd term corresponds to '*external efforts*'.

$$\boldsymbol{\zeta} \;=\; \overline{\overline{\boldsymbol{C}_f}}\,\boldsymbol{\kappa} \qquad \text{(Eq. 5)}$$

The fiber bending energy term, $'\,\boldsymbol{\zeta}\cdot\delta\boldsymbol{\kappa} = \overline{\overline{\boldsymbol{C}_f}}\,\boldsymbol{\kappa}\,\delta\boldsymbol{\kappa}\,'$ can be written in 3D as follows (Eq. 6):

$$\begin{aligned}\left[\overline{\overline{C_f}}\,\boldsymbol{\kappa}\,\delta\boldsymbol{\kappa}\;\right]_{3D} &= A\frac{\partial^2 u}{\partial x^2}\delta\left(\frac{\partial^2 u}{\partial x^2}\right) + B\frac{\partial^2 u}{\partial y^2}\delta\left(\frac{\partial^2 u}{\partial y^2}\right) + C\frac{\partial^2 u}{\partial z^2}\delta\left(\frac{\partial^2 u}{\partial z^2}\right) + D\frac{\partial^2 v}{\partial x^2}\delta\left(\frac{\partial^2 v}{\partial x^2}\right) + E\frac{\partial^2 v}{\partial y^2}\delta\left(\frac{\partial^2 v}{\partial y^2}\right) + F\frac{\partial^2 v}{\partial z^2}\delta\left(\frac{\partial^2 v}{\partial z^2}\right)\\ &+G\frac{\partial^2 w}{\partial x^2}\delta\left(\frac{\partial^2 w}{\partial x^2}\right) + H\frac{\partial^2 w}{\partial y^2}\delta\left(\frac{\partial^2 w}{\partial y^2}\right) + I\frac{\partial^2 w}{\partial z^2}\delta\left(\frac{\partial^2 w}{\partial z^2}\right) + J\frac{\partial^2 u}{\partial x\partial y}\delta\left(\frac{\partial^2 u}{\partial x\partial y}\right) + K\frac{\partial^2 u}{\partial x\partial z}\delta\left(\frac{\partial^2 u}{\partial x\partial z}\right) + L\frac{\partial^2 u}{\partial y\partial z}\delta\left(\frac{\partial^2 u}{\partial y\partial z}\right) \qquad \text{(Eq. 6)}\\ &+M\frac{\partial^2 v}{\partial x\partial y}\delta\left(\frac{\partial^2 v}{\partial x\partial y}\right) + N\frac{\partial^2 v}{\partial x\partial z}\delta\left(\frac{\partial^2 v}{\partial x\partial z}\right) + O\frac{\partial^2 v}{\partial y\partial z}\delta\left(\frac{\partial^2 v}{\partial y\partial z}\right) + P\frac{\partial^2 w}{\partial x\partial y}\delta\left(\frac{\partial^2 w}{\partial x\partial y}\right) + Q\frac{\partial^2 w}{\partial x\partial z}\delta\left(\frac{\partial^2 w}{\partial x\partial z}\right) + R\frac{\partial^2 w}{\partial y\partial z}\delta\left(\frac{\partial^2 w}{\partial y\partial z}\right)\end{aligned}$$

Where, $A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R$ are the non-local mesostructure material parameters. Berkache et al. [13] has proposed a methodology to identify these parameters with respect to Representative Volume Element (RVE) of woven materials.

*Principle of Virtual work and Equilibrium Equations (2D):*

In this part, the development is proposed just for 2D case. For the convenience of the derivation, curvatures ($\boldsymbol{\kappa} = \kappa_{ijk}$) is written as:

$$\frac{\partial^2 u}{\partial x^2} = u_{1,ii} \; ; \; \frac{\partial^2 u}{\partial y^2} = u_{1,jj} \; ; \; \frac{\partial^2 v}{\partial x^2} = u_{2,ii} \; ; \; \frac{\partial^2 v}{\partial y^2} = u_{2,jj} \; ; \; \frac{\partial^2 u}{\partial x \partial y} = u_{1,ij} \; ; \; \frac{\partial^2 v}{\partial x \partial y} = u_{2,ij} \qquad \text{(Eq. 7)}$$

Where, i and j, with (,) denotes the order of partial differentiation. Let $\delta u_k$ $(k = 1,2)$ and $\delta u_{k,ij}(k = 1,2)$ be the virtual displacements, and $\delta\varepsilon_{ij}$ , $\delta\kappa_{ijk}$ be the associated virtual strains. Thus, the Internal Virtual Work (IVW) of 2D composite in the framework of plane stress and assumption of small strain, $\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$ is given by (Eq. 8):

$$IVW = -\iint_{\omega \to 2D} S_{ij} \cdot \delta\left\{\frac{1}{2}\left(u_{i,j} + u_{j,i}\right)\right\} d\omega - \iint_{\omega \to 2D} \zeta_{ijk} \cdot \delta\left\{\begin{matrix} u_{1,ii} \;, \; u_{1,jj} \;, u_{1,ij}, \\ u_{2,ii} \;, u_{2,jj} \;, \; u_{2,ij} \end{matrix}\right\} d\omega \qquad \text{(Eq. 8)}$$

After performing integration by parts of both the terms in the above equation and using the principle of virtual work (PVW) for case of static analysis, the equilibrium equations (in weak form) of the homogenized nonlocal theory for analysis of compressive strength of the composite in 2D can be written as follows (Eq. 9):

$$\Pi = IVW + EVW = 0 \qquad \forall \delta\boldsymbol{u} \qquad \text{(Eq. 9)}$$

where, External Virtual Work (EVW) is as follows (Eq. 10):

$$EVW = -\left\{\begin{matrix} \iint_{\omega} F_k \cdot \delta u_k \, d\omega + \oint_S f_k \cdot \delta u_k \, dS + + \sum P \, \delta u_k \; + \\ \sum \iint_{\omega} M_k \cdot \delta u_{k,i} \, d\omega + \sum \oint_S m_k \cdot \delta u_{k,i} \, dS + \oint_S C_\tau \cdot \frac{\partial \delta u_k}{\partial n} \, dS \end{matrix}\right\} \qquad \text{(Eq. 10)}$$

with $k = 1,2$ and $\gamma = i \;\; or \;\; j$,

where, $F_k$ is Body/Volume force; $f_k$ is Traction/surface force; $M_k$ is Couple force on the plane/body; $m_k$ is Couple force on the edge/surface of the plane/body; $C_\tau$ is Couple spread on the edge of the plane/body, only normal at the edge of plate and $P$ is contributions of concentrated load at the corner/edge. Consequently, the local equilibrium equations in strong form and the boundary conditions can be written as follows (Eq. 11):

Equilibrium equations (Strong Form):

a) *Domain*$(\omega)$:

$$S_{ij,j} = -F_k \quad \text{and} \quad \zeta_{ijk,ji} = -\sum M_k \qquad \forall point \;\; on \; \in \; \omega$$

*b) Boundary(S):*

$$-S_{ij} \cdot n_j = -f_k \;\; ; \;\; \zeta_{ijk} \cdot n_\gamma \cdot n_\gamma = -C_\tau \;\; ; \;\; \zeta_{ijk,j} \cdot n_\gamma + \frac{\sum_{i=1}^{8} \partial\zeta_{ti}}{\partial S} = -\sum m_k$$

$$\forall point \;\; on \; \in \; S \qquad \text{(Eq. 11)}$$

Boundary conditions (BC) on S:

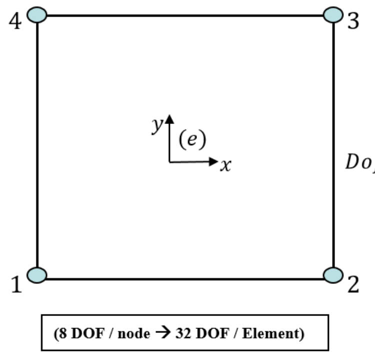
Specify: $\quad i) \quad u_k \quad or \quad S_{ij} \cdot n_j \quad ; \quad ii) \quad u_{k,i} \quad or \quad \zeta_{ijk} \cdot n_\gamma \cdot n_\gamma$

where, $k = 1,2 \;\; and \;\; \gamma = i,j$.

**2.2 Finite element part (Formulation of 2D Non-local element: NL U32)**

The Lagrange elements (available in ABAQUS ©) are $C^0$ type continuous elements. The solution field variable (for ex: displacement, $U$), which is being approximated in Lagrange type of elements are only continuous between element, but not their derivatives. In order to solve the formulation of microbuckling problem using FEM accurately, it is must to have a $C^1$ type continuous element (Hermit type), in order to capture the distribution of displacement and micro rotation of fibre at mesoscopic level.

It is also important to note that, the strain gradient quantities are the second order spatial derivatives of displacement. If we use a reference element, it is necessary to link the first and second order derivatives between two coordinate systems (global and local). Consequently, interpolation of geometry should permit to establish this link (Jacobi for first and second derivate). Therefore, a new non-local super parametric element (NL U32) is formulated, similar to Bogner-Fox-Schmit rectangle [14] as shown in (Fig. 2), where the degrees of freedom (D.O.F) of the element is not only the displacements, but also their derivatives.




$$Dof, \boldsymbol{d}^{(e)} = \begin{Bmatrix} u \\ \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial^2 u}{\partial x \partial y} \\ v \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial^2 v}{\partial x \partial y} \end{Bmatrix} = \begin{Bmatrix} u \\ u'_x \\ u'_y \\ u''_{xy} \\ v \\ v'_x \\ v'_y \\ v''_{xy} \end{Bmatrix}$$

*Fig. 2. Non-Local Super-parametric Element (NL U32)*

- **Interpolation functions:**

a) Displacement :

$$\boldsymbol{u}_h(\xi,\eta) \cong \sum_{i=1}^{16} N_i^{(e)}(\xi,\eta)\,\overline{\boldsymbol{u}}_i \quad ; \quad \boldsymbol{v}_h(\xi,\eta) \cong \sum_{i=1}^{16} N_i^{(e)}(\xi,\eta)\,\overline{\boldsymbol{v}}_i$$

i) Complete 3rd order cubic polynomial is chosen;
ii) 1D cubic Hermit type polynomials are used to build 16 displacement interpolation functions ($N_i$). Where, $\xi$ and $\eta$ are the local coordinates.

b) Geometry :

$$x \cong \sum_{j=1}^{9} N_j^{(e)}(\xi,\eta)x_i \quad ; \quad y \cong \sum_{j=1}^{9} N_j^{(e)}(\xi,\eta)y_i$$

i) Complete 2nd order biquadratic polynomial is chosen to build 9 interpolation functions ($N_j$) for geometry in order to calculate the second derivatives in Jacobi, as discussed earlier.

The numerical integration is performed by using (4x4) Gauss quadrature rule. At present, the element is developed in order to take into account a non-linear behavior of stress and micro moment. User material subroutine, UMAT is associated with UEL subroutine which permits to integrate hardening of matrix or damage. The geometrical non-linearity requires addition of the classical terms:

$$- \iint_{\omega \to 2D} S_{ij} \cdot \left\{ u_{i,j} \delta u_{j,i} \right\} + S_{ij} \cdot \left\{ u_{j,i} \delta u_{i,j} \right\} d\omega \qquad \text{(Eq. 12)}$$

## 3. Results

### 3.1 Validation of NL U32 element

In order to compare the accuracy and validate Non-Local element (NL U32) for linear elastic isotropic case, results are compared with ABAQUS © linear plane stress element (CPS4), as it is much more convenient reference element at the moment, since it is also built with plane stress formulation. The values of $\overline{\overline{\boldsymbol{C}_f}}$ parameters $(A, B, C, D, E, F)$ is unknown for NL U32 element. Therefore, it is important to understand the influence of these parameters on the solution. Hence, $\overline{\overline{\boldsymbol{C}_f}}$ parameters value are varied and kept constant in all the cases. Results of only few relevant test cases are presented here. Material properties for CPS4 element: Elastic Young's modulus, $E = 2.0E5\ MPa$, Poisson's ratio, $\nu = 0.3$ and thickness, $t = 1mm$. For NL U32 element, material properties are: Matrix Young's modulus, $E_m = 2.0E5\ MPa$, Poisson's ratio, $\nu_m = 0.3$, thickness, $t = 1mm$, the values for parameters of local fiber bending stiffness matrix $A, B, C, D, E, F$: started with initial guess of low value, $0.15\ MPa \cdot mm^2$ and varied with the order of 100. Analysis is performed on a rectangular plate (100 x 50 mm), meshed with 276 Elements (312 nodes).

**Test Case:** Compression

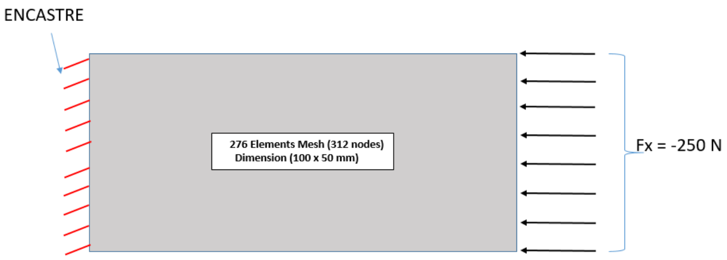



*Fig. 3. Mesh, Load and Boundary conditions*

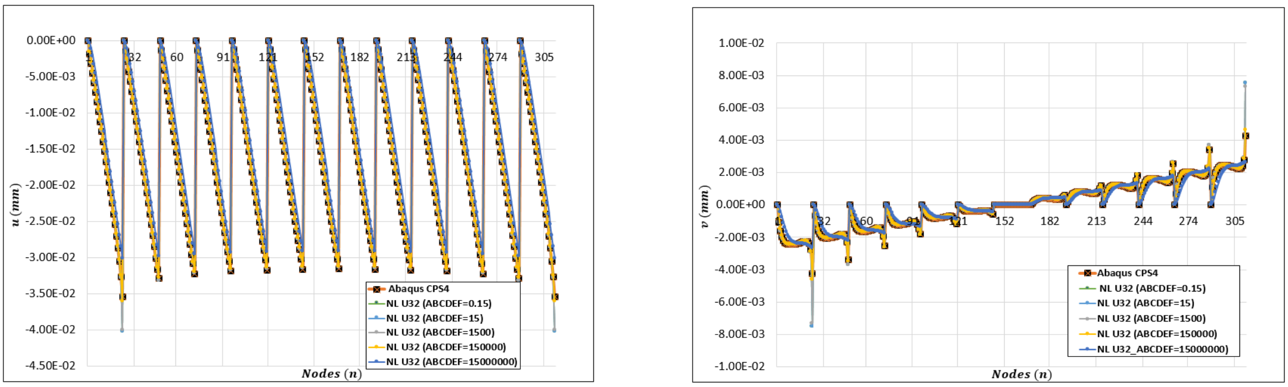
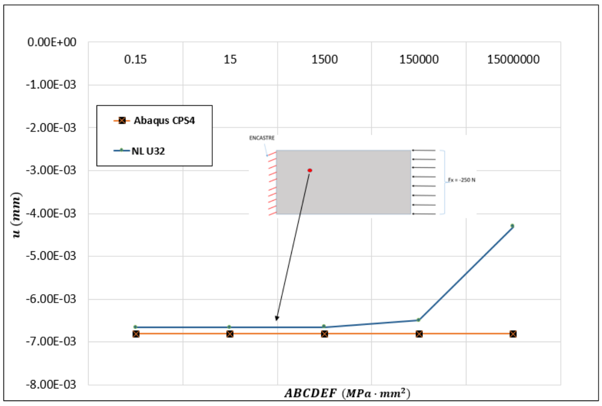



*Fig. 4. Influence of Cf Parameters*



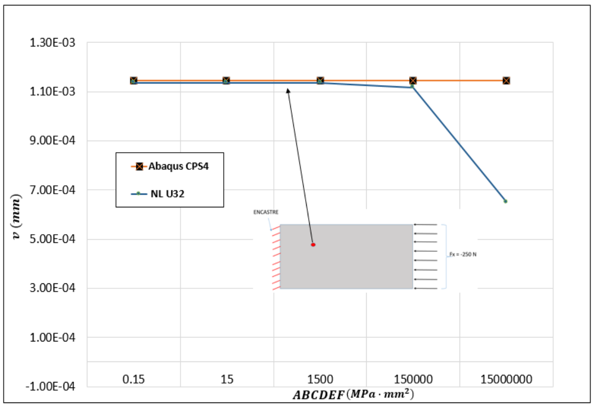




*Fig. 5. Comparison of Displacements (u,v) at node 222 for different Cf parameters value*

From (Fig. 4), it can be observed that, similar response and difference in solution is very less compared to ABAQUS © CPS4 element when local fiber bending stiffness matrix, $\overline{\overline{\boldsymbol{C_f}}}$ parameters (ABCDEF) varied up to 1500 $MPa \cdot mm^2$. But, significant difference in solution is observed compared to CPS4 element when ABCDEF value is increased above 1500 $MPa \cdot mm^2$ (see Fig. 5). Much stiffer response can be observed with NL U32 compared to continuum solid CPS4. This is because, for lower order of $\overline{\overline{\boldsymbol{C_f}}}$ ($\leq 10^3$), the order of bending energy ($\boldsymbol{\kappa}^T \overline{\overline{\boldsymbol{C_f}}}\, \boldsymbol{\kappa}$) becomes lower compared to classical strain energy ($\boldsymbol{\varepsilon}^T\, \overline{\overline{\boldsymbol{D}}}\, \boldsymbol{\varepsilon}$), since the order of curvatures is also lower. Consequently, bending energy has negligible contribution to the total energy. So, we tend to obtain similar solution as classical plane stress solution. But, for higher order of $\overline{\overline{\boldsymbol{C_f}}}$ ($> 10^3$), the order of bending energy becomes similar or higher compared to the classical strain energy. Consequently, we can observe significant difference in solution for ABCDEF >1500 $MPa \cdot mm^2$. The level of stiffness which generates an effect, can be estimated (but not shown here), and we have also obtained the value of 1500 $MPa \cdot mm^2$. In conclusion, in order to have non-local effects in this particular solution, it is necessary to have $\overline{\overline{\boldsymbol{C_f}}}$ parameters value $> 10^3$. This clearly explains the influence of $\overline{\overline{\boldsymbol{C_f}}}$ parameters value on the solution. In future, it is necessary to obtain proper values for ABCDEF in correspondence to a composite mesostructure. It is also important to note that, the choice of interpolation functions, integration rule for both ABAQUS © CPS4 and NL U32 elements are different, which can also cause difference in solution, especially near the points of loading. However, the global response obtained with NL U32 is almost similar compared to ABAQUS © CPS4 element, which validates the non-local element for linear isotropic elastic case.

*Mesh Convergence study*:

In order to evaluate the convergence of the solution obtained with NL U32 element against mesh size, various mesh size of: $no. of\ elements, n = 8, 32, 66, 128, 276, 512$ is chosen. The material properties are same as previous case, except that the two values of ABCDEF is chosen: 15 $MPa \cdot mm^2$ and 1500000 $MPa \cdot mm^2$ . The solution is compared with ABAQUS © CPS4 element. We can observe from (Fig. 6), that as the mesh size (n) is increased, we tend to obtain an asymptotic or converged solution for both NL U32 element and ABAQUS © CPS4 element. As discussed earlier, for lower values of ABCDEF with increase in mesh size, we tend to obtain similar solution as classical plane stress element CPS4 and for higher values of ABCDEF, quite significant difference can be observed. However, the convergence is obtained with increase in mesh size. This confirms that for finer mesh, we will obtain a good converged solution with NL U32 element.

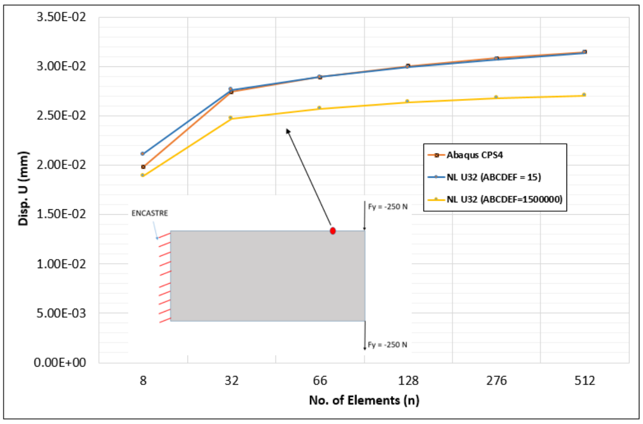



*Fig. 6. Mesh Convergence study of NL U32 element*

### 3.2 Comparison with 2D heterogeneous complete microstructure

As a final step of validation of Non-Local Homogeneous model implemented in non-local element (NL U32) for linear elastic case, results are compared with 2D Unidirectional (UD) composite (T300/914 Carbon/epoxy) model built using ABAQUS © plane stress element (CPS4). The stacking sequence of UD heterogeneous composite (with 10 layers) model built in ABAQUS © is as shown in the (Fig. 7). The elastic properties of T300/914 Carbon/epoxy UD ply obtained by [15] as in (Tab. 1) is assigned for matrix and fiber in both the models.

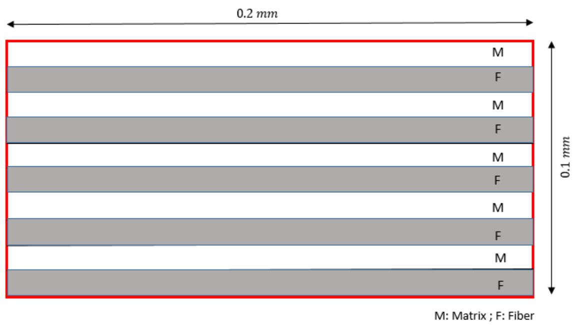



*Fig. 7. 2D composite stacking sequence: UD plies at* $0^0$

| **Heterogeneous model Abaqus (CPS4)** | | **Homogenous Non-local model (NL U32)** | |
|---|---|---|---|
| matrix | fiber | matrix | fiber |
| $E_m = 4500\ Mpa$ | $E_f = 24000\ Mpa$ | $E_m = 4500\ Mpa$ | $E_f = 24000\ Mpa$ |
| $\nu_m = 0.4$ | $\nu_f = 0.3$ | $\nu_m = 0.4$ | $\nu_f = 0.3$ |
| thickness, $t = 1mm$ | | Volume fraction, $f = 0.625$ | |
| | | Diameter of fibers, $d_f = 0.01mm$ | |
| | | Local fiber bending stiffness parameter for UD ply, $D = fE_f r_{gf}^2 = 0.09375\ MPa \cdot mm^2$ | fiber gyration radius, $r_{gf} = \sqrt{\frac{I}{S_f}}$ |
| | | ABCEF = 0.001*D | thickness, $t = 1mm$ |

*Tab. 1. Material parameters of UD ply [15]*

It is important to note that, for UD ply, since there is just one gradient ($v'' = \frac{\partial^2 v}{\partial x^2}$) along thickness direction, as in case of [2], just one parameter (D) of local bending stiffness matrix, $\overline{\overline{\boldsymbol{C}_f}}$ is considered, values for other parameters (ABCEF) is assigned very low compared to parameter D in homogenous non-local model.

*Results:*

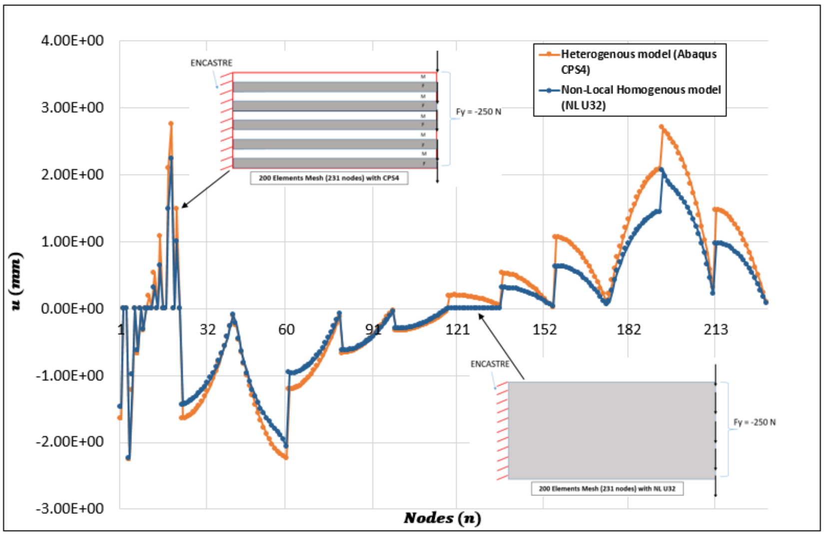



*Fig. 8. Comparison of variation of displacement (u) over nodes*

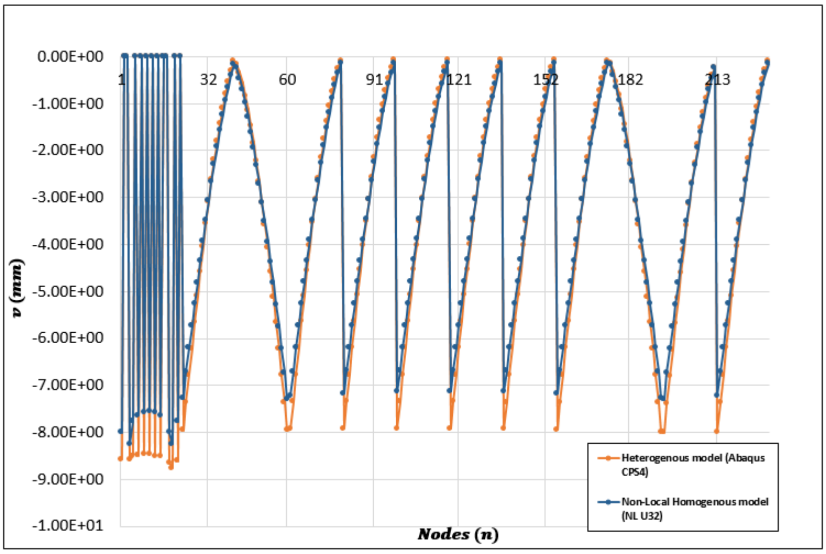



*Fig. 9. Comparison of variation of displacement (v) over nodes*

From (Fig. 8) and (Fig. 9), it is evident that the results (displacements: $u, v$) obtained from homogenous non-local model is in close comparison with the results of heterogeneous model of ABAQUS © for the case of UD composite subjected to bending. The slight difference between two solutions are clearly due to the addition of non-local terms and choice of interpolation functions (for displacement and geometry) in homogenous non-local model. With this validation, it can be concluded that the homogenized non-local model (implemented in NL U32 element) can be used for analysis of UD composites (linear isotropic elastic cases).

## 4. Conclusion

A homogenized non-local finite element model has been proposed, similar to Mindlin' second gradient theory [1] and the first numerical developments have been initiated. The framework of this nonlocal modeling is more general that of [2] to assess microbuckling phenomenon in UD and complex composites (carbon/epoxy long fiber) at the structural/mesoscopic scale. The developed non-local numerical model is implemented in User Element (UEL) subroutine for analysis in ABAQUS ©, thereby developing a 2D $C^1$ continuous non-local super-parametric element (NL U32) for linear isotropic elastic case. Various tests results are presented to validate the capability of NL U32 element in comparison with ABAQUS © iso-parametric plane stress element (CPS4) and also to understand the influence of $\overline{\overline{\boldsymbol{C}_f}}$ parameters. In addition, as a final check for validation, the non-local homogenous 2D model implemented in NL U32 is compared with complete 2D UD heterogeneous composite structure, modeled using ABAQUS © CPS4 element. From all the test cases, good results are obtained in comparison with the reference chosen, thereby validating the capability of non-local element (NL U32).

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