Analyse globale-locale de structures composites

Global-local analysis of composite structures

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Résumé

Le processus de conception et de certification des structures aérospatiales nécessite une caractérisation détaillée de l'état de contraintes. Toutefois, la complexité de ces structures et l'utilisation de matériaux composites peuvent considérablement augmenter le temps de calcul des modèles. Ce travail propose une méthodologie de modélisation de type *global-local* pour mettre en place un modèle d'ordre élevé uniquement pour une région réduite du modèle global. La méthodologie a été mise en œuvre en deux étapes. La première étape est consacrée à l'analyse statique sur un modèle global de la structure et se fait à l'aide d'un logiciel commercial de calcul par éléments finis utilisant des éléments standard 1D / 2D. Dans une deuxième phase, un modèle 1D d'ordre élevé est créé localement dans la zone d'intérêt (ZOI), en important les informations de l'analyse globale précédente. Les théories raffinées utilisées dans l'analyse détaillée sont mises en œuvre dans le cadre de la formulation unifiée proposée par Carrera. En raison des caractéristiques de ces théories d'ordre élevé, il est possible de saisir précisément l'état de contrainte 3D et de pouvoir bien décrire des phénomènes complexes telles que les effets de bord libre dans des structures composites. Des benchmarks seront illustrés pour démontrer l'efficacité de la méthodologie proposée.

Abstract

The design and certification process of aerospace structures requires a detailed stress characterization. Nevertheless, the complexity of the aircraft structures and the use of composite materials can significantly increase the computational costs of the models. This work proposes a global/local modelling strategy to set up an high-order model only for a reduced region of the global model. The methodology has been implemented in two steps. The first step is devoted to the static analysis on a global model of the structure and it is done by means of a commercial Finite Element (FE) software using 1D/2D standard elements. In the second step, an high-order beam model is locally built, i.e. within the zone of interest (ZOI), importing the information from the previous global analysis. The refined theories utilized in the detailed analysis are implemented in the framework of the Carrera Unified Formulation (CUF) and are not available in commercial software. Due to the characteristics of the CUF refined beam theories, complex 3-D stress fields are detected and complex phenomena as the free edge effects in composite structures, are easily studied with the proposed global/local modelling approach. Some benchmarks are reported to prove the effectiveness of the proposed methodology.

Mots Clés : Analyse globale / locale, méthode des éléments finis, théories d'ordre supérieur, formulation unifiée de Carrera
Keywords : Global/Local Analysis, Finite Element Method, Refined Beam Theories, Carrera Unified Formulation

1. Introduction

In the aeronautical field, when dealing with the design of an aircraft structure the finite element (FE) model of the system is usually built by combining 1D and 2D elements, which opportunely discretize mathematical domains of stringers, panels, ribs, and other components. Clearly, this discretization is a simplification of the reality. Moreover the use of the composite materials for the aircraft structure components compromises any kind of predictions about the structural behaviours if 3D stress fields can not been detected in the proximity of holes, edges, connections and thickness discontinuities in the panels. To accurately capture these localized 3D stress fields, solid models or high-order theories are often necessary. However, in order to make the model more efficient, i.e. to balance computational cost and results accuracy, a global/local approach is often employed. In the last decades, several approaches have been developed in the literature to deal with the global/local analysis for composite materials, see [1]. Among these methods, particular attention is dedicated to the so-called ‘multi-steps methods’, in which the local analysis of the zone of interest (ZOI) requires the boundary conditions (BCs) at the interface level that are obtained by the analysis on the global structure. For instance, the global/local methodology proposed by Mao *et al.* [2] makes use of a coarse mesh to analyse the entire structure and to obtain the nodal displacements which were subsequently applied as BCs to the refined local model. A two-steps methodology for the global/local stress analysis is shown in the work of Ransom and Knight [3]. The first step is devoted to the analysis of the global low-fidelity (LF) model and to the BCs computation for the local high-fidelity (HF). Spline interpolation functions are used to compute the displacements and rotations in the nodes of the local mesh from those resulting from the global model. The local analysis is done in a second step and it is completely independent of the global one and their results prove that the methodology can be used to detect complex stress fields in restricted areas of the structure with a considerable computational cost reduction. The work of Thompson *et al.* [4] is one of the first examples of global/local analysis from a 2D global model to 3D local one. The authors realised a 2D global model of the laminate composite plate making the use of a zooming technique to refine the mesh in the proximity of the hole to avoid the displacements interpolation in the interfaces between global/local 3D model. In the framework of the Carrera Unified Formulation (CUF), a first example of global/local analysis was the use of the Arlequin method to couple 1D finite elements differing in the approximation order of the displacement field [5] and similar results were reproduced by Carrera et al. [6] by coupling models with different kinematics by using point-wise Lagrange multipliers. Recently, CUF has been extended in [7] to deal with the global/local analysis of laminates by employing its intrinsic variable-kinematics capability. This work proposes a global/local methodology that consists of a two-step procedure for the evaluation of accurate stress fields in critical regions of structures. In the proposed method, the first step is devoted to the static analysis of a global model of the structure and it could be done by commercial software using 1D/2D elements. A criterion is established to identify the most critical region, which is subsequently analyzed in the second step by using high-order models, to obtain accurate stress fields. The refined theories used in the detailed analysis are implemented in the CUF framework which is a very effective FE tool for evaluating complex strain/stress fields of composite structures [1, 8, 9]. Thanks to the proposed global/local methodology, the information of the static analysis on a global model composed 1D/2D elements, can be exploited to obtain a detailed description of the stress field using high-order beam theories in the CUF framework in critical region of the structure on which suitable BCs (derived from the global LF FE model) are imposed, at reduced computational costs.

2. Global/local modelling approach

Generally, FE modelling of composite aircraft structures is performed by using a combination of 1D (stringers, spar caps) and 2D (skin, ribs) finite elements. Although these models are affected by geometrical inconsistency, because the employed 1D and 2D elements may have incompatible kinematics and the use of fictitious links is usually required, they can provide reliable solutions and accurate results at global scale. As a matter of fact, these simplified models can only give a good estimation of the global structural behaviour in terms of displacement field. Nevertheless, if an accurate description of the stress field is needed, e.g. in the regions close to holes, free edges or connections, detailed analyses which usually employ 3D finite elements are necessary. However, global/local approaches based can be used instead of 3D elements to make the analysis computationally efficient.

The global/local modelling approach here presented consists of two-steps. The first step involves the analysis of the global model to identify the critical region, i.e. the zone of interest (ZOI), by using a criterion that is established by the designer to extract a proper set of BCs to be applied to the local model. The global is composed of 1D, 2D or a combination of 1D and 2D elements and it can be easily built into a commercial software. Once the BCs are retrieved from the global model and the ZOI has been identified, the second step consists of building the local HF CUF model to carry out the detailed analysis on a pertinent sub-model of the structure.

2.1. Local 1D models based on Unified Formulation

The HF local analysis is performed by using a 1D refined structural theory based on CUF. According to CUF, the layer-wise (LW) displacement field of the composite beam is written as :

$$\mathbf{u}(x,y,z) = F_\tau(x,z)\,\mathbf{u}^k_\tau(y)\,, \quad \tau = 1,2,...,M\,, \tag{Eq. 1}$$

where y is the longitudinal direction of the refined 1D element ; (x,z) are the cross-section coordinates ; $\mathbf{u}(x,y,z)$ is the three-dimensional displacement field ; $\mathbf{u}^k_\tau(y)$ is the vector of generalized displacements for the k-th layer ; and $F_\tau(x,z)$ are the expansion functions of the cross-sectional domain. The class and number of expansion functions is arbitrary, being M the maximum number of expansions, which is a user defined parameter. Repeating indexes denote summation. LW models can be implemented by using Lagrange Expansions (LE). This beam theory, introduced by Carrera and Petrolo [10], is based on the use of interpolating Lagrange polynomials as expansion functions F_τ of the cross-sectional coordinates. In this manner, the cross-section of the composite beam can be discretized with an arbitrary number of Lagrangian domains, which are used to represent the surfaces of each layer. On the other hand, the discretisation of the structure along the y-axis is done by adopting a classical FE formulation. Using standard 1D interpolations, the generalized displacement vector $\mathbf{u}^k_\tau(y)$ of Eq. Eq. 1 can be approximated by the nodal shape functions $N_i(y)$.

$$\mathbf{u}^k_\tau(y) = N_i(y)\,\mathbf{u}^k_{i\tau}\ , \quad i = 1,...,n_n, \tag{Eq. 2}$$

where $N_i(y)$ stands for the $i-th$ shape function, n_n is the number of nodes in one element and $\mathbf{u}^k_{i\tau}$ is the vector of nodal unknowns for k-th layer. For the sake of brevity, the shape functions are not reported here. They can be found in classical books [11]. Elements with four nodes (B4) are adopted in this work, in this way a cubic approximation along the y-axis is assumed. The governing equations for a static problem are derived by applying the Principle of Virtual Displacements (PVD). Considering the stresses, $\boldsymbol{\sigma}$, and strains, $\delta\boldsymbol{\varepsilon}^T$, the virtual variation of the internal work δL_{int} can be expressed as :

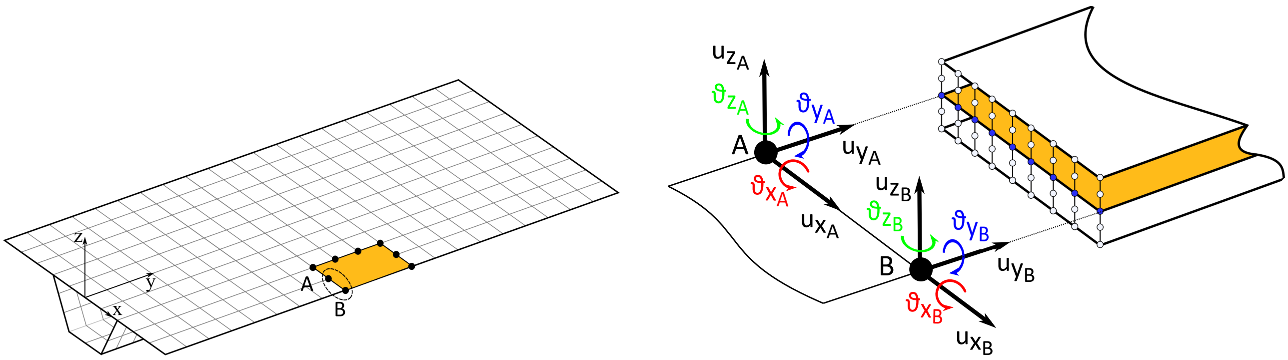
$$\delta L_{int} = \int_V \delta\boldsymbol{\varepsilon}^T\,\boldsymbol{\sigma}\,dV = \delta\boldsymbol{u}^T_{sj}\mathbf{K}^{ij\tau s}\boldsymbol{u}_{\tau i}\,, \tag{Eq. 3}$$

where $\delta\mathbf{u}^T_{js}$ is the virtual variation of the vector of nodal unknowns and $\mathbf{K}^{ij\tau s}$ is the stiffness matrix in the form of a 3×3 fundamental nucleus (FN). The layer index k has been omitted in Eq. 3 for clarity reasons. The derivation of the FN is not reported here, but it is described in [9]. The terms $K^{ij\tau s}_{xx}$ and $K^{ij\tau s}_{xy}$ are provided in the following :

$$\begin{aligned}
K^{ij\tau s}_{xx} = & +\tilde{C}_{22}\int_L N_iN_j dy \int_\Omega F_{\tau,x}F_{s,x}d\Omega + \tilde{C}_{44}\int_L N_iN_j\,dy\int_\Omega F_{\tau,z}F_{s,z}d\Omega + \\
& +\tilde{C}_{26}\int_L N_iN_{j,y}dy\int_\Omega F_{\tau,x}F_s d\Omega + \tilde{C}_{26}\int_L N_{i,y}N_j\,dy\int_\Omega F_\tau F_{s,x}d\Omega + \\
& +\tilde{C}_{66}\int_L N_{i,y}N_{j,y}dy\int_\Omega F_\tau F_s\,d\Omega\,, \\
K^{ij\tau s}_{xy} = & +\tilde{C}_{23}\int_L N_iN_{j,y}dy\int_\Omega F_{\tau,x}F_s d\Omega + \tilde{C}_{45}\int_L N_iN_j dy\int_\Omega F_{\tau,z}F_{s,z}d\Omega + \\
& +\tilde{C}_{26}\int_L N_iN_j dy\int_\Omega F_{\tau,x}F_{s,x}d\Omega + \tilde{C}_{36}\int_L N_{i,y}N_{j,y}dy\int_\Omega F_\tau F_s d\Omega + \\
& +\tilde{C}_{66}\int_L N_{i,y}N_j dy\int_\Omega F_\tau F_{s,x}d\Omega\,,
\end{aligned} \tag{Eq. 4}$$

where $\tilde{C}_{22}$, $\tilde{C}_{23}$, $\tilde{C}_{26}$, $\tilde{C}_{36}$, $\tilde{C}_{44}$, $\tilde{C}_{45}$, $\tilde{C}_{66}$, are the components of the rotated stiffness matrix of the orthotropic material. The comma denotes partial derivatives. All the components of $\mathbf{K}^{ij\tau s}$ can be derived from Eq. 4 by permutations. Furthermore, it should be noted that the formal expressions of the components of the fundamental nucleus $\mathbf{K}^{ij\tau s}$ of the stiffness matrix do not depend on the choice of the cross-sectional functions F_τ, which determine the theory of structure, and shape functions N_i, which determine the numerical accuracy of the FEM approximation. This means that any classical or high-order beam element can be automatically formulated by opportunely expanding the fundamental nuclei according to the indexes τ, s, i, and j. If compared to 3D modelling, CUF has been demonstrated to provide extremely accurate solutions for composite with at least one order of magnitude less of degrees of freedom, see [12].

2.2. BCs application and coupling effects



(a) Global and local regions (b) Global/local coupling

Fig. 1. Global/local analysis in a plate-like model

As discussed above, the first step is the analysis carried out on the global LF FE model. Given the solution from the global model, the displacement variables at the boundary of the local FE model are opportunely transferred in order to meet the CUF formalism. CUF is the ideal tool for global/local analysis for its intrinsic variable kinematics characteristics. In fact, the nformation exchange between global and local FE models can be done easily by employing linear shape functions to determine all the middle plane nodes. The reason of the use of linear shape functions is to maintain conformity with the kinematics of the global model. Furthermore, such an interpolation procedure allows the use of different size for global and local meshes, which are incompatible at the interface. Therefore, Timoshenko's displacement kinematics (if the global model is 1D) or Reissner - Mindlin displacement kinematics (if the global model is 2D) are applied to compute displacements of each node at the interface along the thickness direction. This procedure ensures the minimum information loss and the consistency of the local solution.

For representative purposes, (Fig. 1) shows an example of global/local analysis for a 2D stringer model at free edge. According to Mao *et al.* [2], the application of the BCs in the local region unavoidably introduces errors and to minimize the effect of such errors a local refinement of the mesh is adopted to confine the detrimental effects of the BCs application into a reduced zone near the interface of the global/local model where BCs are applied. In (Fig. 2), the representation of the local FE model in the CUF framework is given. The expansion points of the LE domains toward the edges of the ZOI are distributed with the square root of the well-know Chebyshev node formula that permit an high refinement in proximity of the free edge of the model. The formula for the x-axis distribution of nodes is the following :

$$x_k = \sqrt{\cos(\frac{2k-1}{2n}\pi)}, \qquad k = 1, \ldots, n_x. \qquad \text{(Eq. 5)}$$

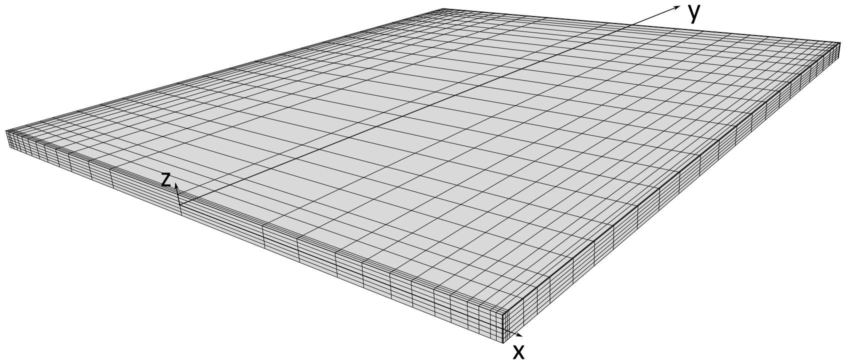


Fig. 2. Local CUF model

Where n_x is a positive integer computed from the number of sub-domains $N_{x_{exp}}$ used for the cross-section of the local CUF model along the x-axis, $n_x = 2 \cdot N_{x_{exp}} - (N_{x_{exp}} - 1)$.

3. Numerical results

Three benchmarks of global/local analyses have been considered to assess the validity of the proposed methodology.

3.1. Cantilever beam

The first benchmark is an isotropic cantilever beam and it is studied to demonstrate the ability of the proposed methodology in detecting an accurate 3D stress field, at a reduced computational cost. The beam has a rectangular cross-section and it is characterized by the following geometric features : width = 1.0 *mm*, height = 10.0 *mm* and length = 90.0 *mm*. An isotropic material is used for the structure with Young's modulus E = 75 GPa and Poisson ratio ν = 0.33. A shear point P load is applied at the free end of the beam and its magnitude is equal −1 N.

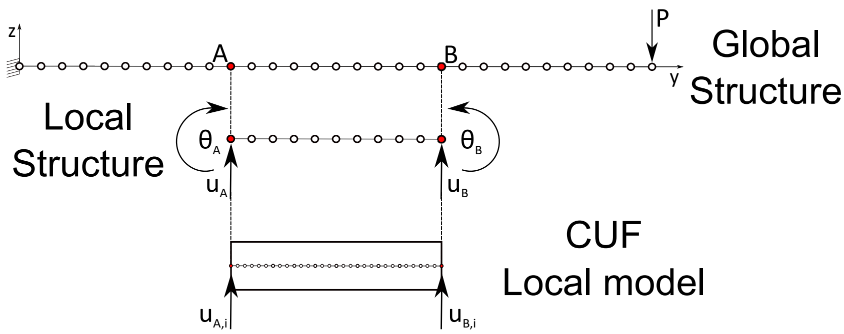


Fig. 3. Global and local models for the cantilever beam

The local region of the structure is shown in (Fig. 3), whose domain extends between points A = [0, 30, 0] mm and B = [0, 60, 0] mm in the global reference system. A 'monolithic' beam model is set in the commercial software MSc Nastran using 30 1D-beam elements. A preliminary static analysis of this model gives the displacements and rotations (geometrical BCs) at the boundary nodes of the local region. In the second step, a CUF local beam model is set for the local region applying the geometrical BCs. In (Fig. 3), the displacements $u_{A,i}$ and $u_{B,i}$ are the generic traslational displacements of the cross section 'i' point. They are computed with Timoshenko's displacement field exploiting the vertical displacements (u_A and u_B) and rotations (θ_A and θ_B) of the static analysis in the global model. Two different CUF models have been used in the global/local approach : 3x3 L9 and 1x5 L16 for the beam cross-section with 10 B4 elements for the structural mesh along y-axis. (Tab. 1) and (Fig. 5) show the comparison between a monolithic 3D global model, obtained by ABAQUS CAE, and the global/local models in terms of the most significant stress components that are the axial stress σ_{yy} and the shear stress σ_{yz}. The distributions of the stresses in (Fig. 5) are computed at $x = 0$, $y_{global} = 45.0$ mm and the stress values, which are reported in (Tab. 1), are at the z coordinate that gives the maximum value of these stress.

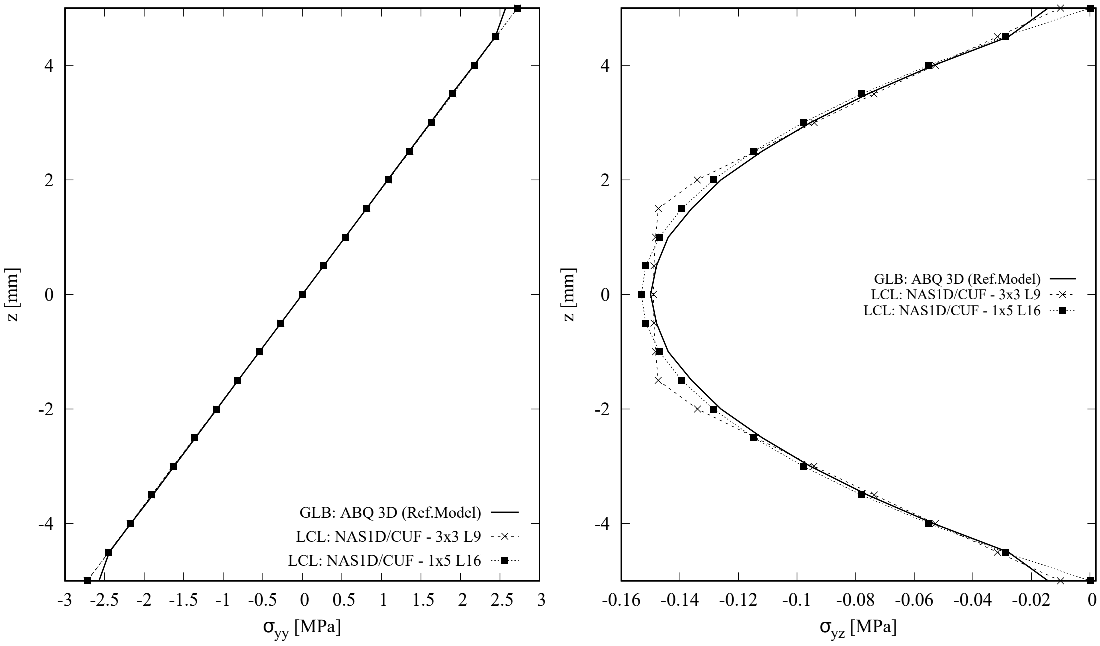


Fig. 4. Axial σ_{yy} and shear σ_{yz} distribution distribution through the thickness

	σ_{yy} $[MPa]$	$\sigma_{yz} \cdot 10^{-1}$ $[MPa]$	DOFs
Monolithic ABAQUS 3D	2.571	-1.481	14209
NASTRAN 1D / CUF - 3x3 L9	2.721	-1.491	546/4557
NASTRAN 1D / CUF - 1X5 L16	2.717	-1.532	546/6054

Tab. 1. σ_{yy} and σ_{yz} for the cantilever beam with a shear point load

All the global/local results are in a good agreement with those of monolithic 3D global model which constitutes the reference solution in this case. The most important achievement is the reduction of the computational cost and the possibility to describe the shear stress trend correctly using higher order beam models.

3.2. Cantilever composite beam under bending

A composite beam with 3 plies is considered as a second benchmark. The considered beam is clamped at one end and free at the other end, and the structure is loaded by a point load along the z-direction at the centre of the free end and its magnitude is $-1 \cdot 10^{-3}$ N as illustrated in (Fig. 5).
The beam has a length of $L = 2.0\ m$ and a width $b = 0.1\ m$. The total thickness is $t = 0.003\ m$, the ply thickness is $t_{ply} = 0.001$ m and the stacking sequence of the laminate is $[0°/90°/0°]$. An orthotropic material is taken into account with the following material properties : $E_{11} = 40\ GPa$, $E_{22} = E_{33} = 4.0$ GPa, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$, $G_{12} = G_{13} = G_{23} = 1.0$ GPa.
The global model is built in *MSc-Nastran* with a mesh of 10 x 200 plate elements.
The ZOI is a small square region within the global structure. It is located at the centre of the model and its geometrical parameters are $L_{local} = 10.0\ mm$, $b_{local} = 10.0\ mm$ $h_{local} = t = 3.0\ mm$. The ZOI is highlighted in (Fig. 5).
In the CUF local model 20 B4 structural beam elements are used along the y-axis direction and two

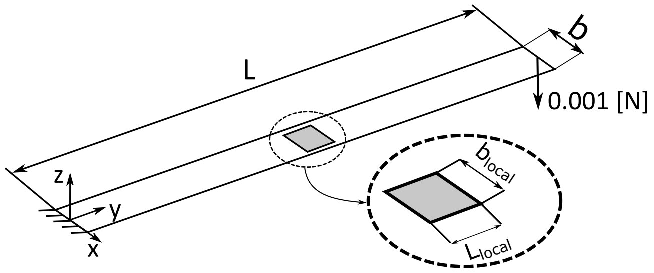


Fig. 5. Geometry and BCs for the cantilever composite beam.

different types of LE sub-domain distributions are adopted. The first one consists of 5x9 L9 sub-domains (5 along x and 9 along z) while the second one consists of 5x9 L16 sub-domains (5 along x and 9 along z) with 3 sub-domains for each layer of the structure.
The results of the static analysis are evaluated at the centre of the local region that is coincident with the global one.
In (Fig. 6), the axial stress σ_{yy} and shear stress σ_{yz} distribution through the thickness are presented. The plot compares the results of the global analysis in *MSc-Nastran* with those of the global/local approach.

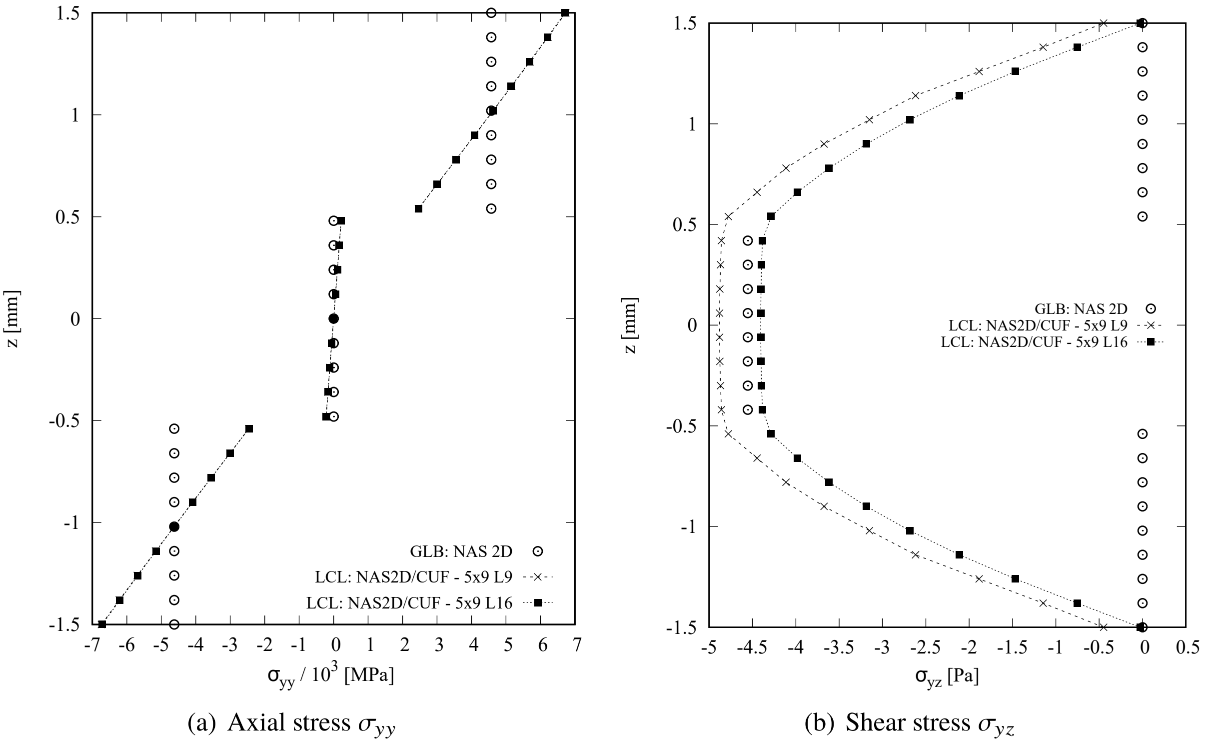


Fig. 6. Axial stress σ_{yy} and Shear stress σ_{yz} along the z-axis for the 3 Ply Composite Plate

For this benchmark, the commercial code always gives constant values of in-plane and out-plane stresses in each layer of the laminate. With the global/local approach, it is possible to detect the real trend of all the stresses. Indeed, as shown in (Fig. 6 a) and (Fig. 6 b), the local HF FE model catches the linear behaviour of the axial stress as well as the quadratic behaviour of the shear stress through the thickness. In particular, the model with 5x9 L16 sub-domains is able to predict the null value of the σ_{yz} at the top and bottom of the cross-section.

3.3. Free edge example

The analysis of a composite coupon made of G947/M18 carbon-epoxy as shown in Fig. 1 is carried out. The laminate has four plies with stacking sequence $[10°/-10°]_s$ and dimensions as follows : length $L = 200$ mm, width $w = 20$ mm, thickness $t = 0.76$ mm. The elastic properties of the ply are given in Lagunegrand et al. [13] ; these are : $E_{11} = 97.6$ GPa, $E_{22} = 8.0$ GPa, $E_{33} = 8.0$ GPa, $\nu_{12} = 0.37$, $\nu_{13} = 0.37$, $\nu_{23} = 0.50$, $G_{12} = 3.1$ GPa, $G_{13} = 3.1$ GPa, $G_{23} = 2.7$ GPa.
The coupon is subjected to uniaxial tension (longitudinal strain is $\epsilon_{yy} = 0.001$). The dimensions of the local region are $w_{local} = 4\ mm$ and $L_{local} = 16\ mm$. The global model is analyzed with MSc-Nastran using CQUAD4 elements. On the other hand, the local region is analyzed using higher-order CUF models with LW capabilities. (Fig. 7 a) shows the distribution of the transverse shear stress (σ_{yz}) at

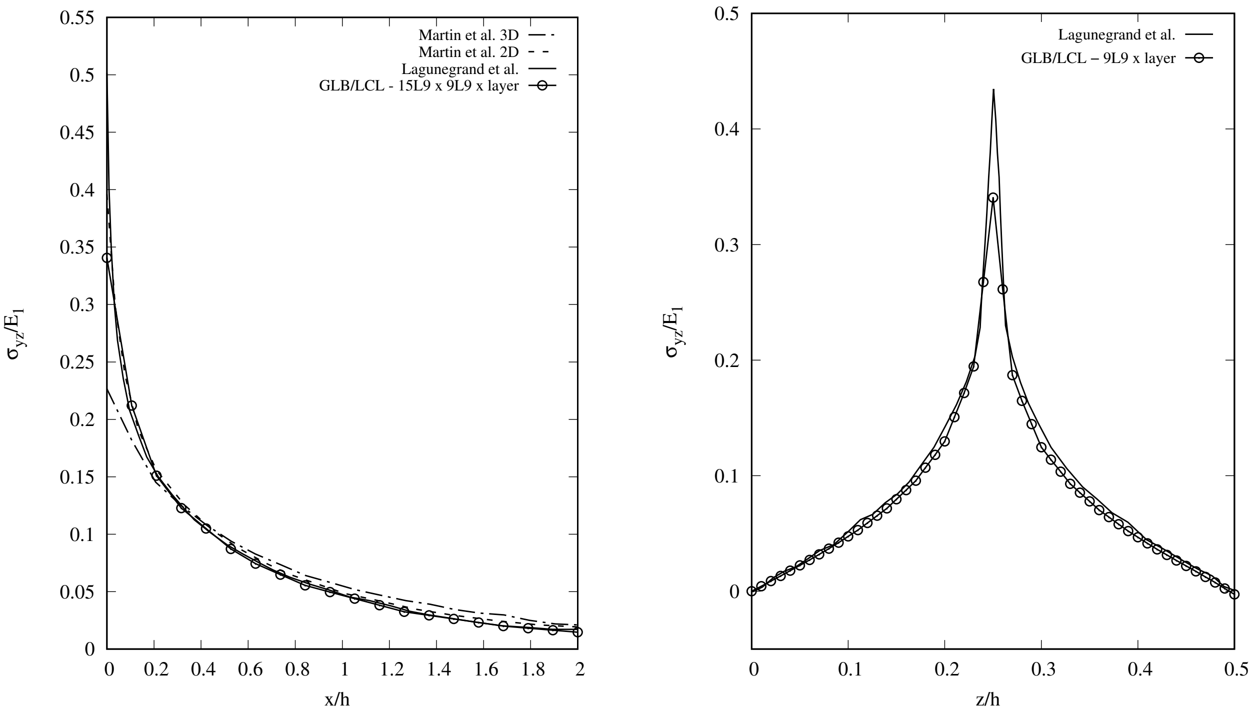


Fig. 7. Interlaminar stress distribution at free edge

the interface $-10°/10°$ along axis x in the local region ($x = 0$ is the free edge). Furthermore, (Fig. 7 b) shows the through-the-thickness distribution of the of the same stress component at the free-edge. The proposed results are compared with those given by Lagunegrand et al. [13] and Martin et al. [14]. The provided methodology is able to deal with free-edge effects and stress singularities when compared to other approaches taken from the literature.

4. Conclusions

In this paper, a two-step global/local modelling approach has been developed for global/local stress analysis in the CUF framework. In the first step, a preliminary static analysis on the 1D/2D model by using commercial software is done for obtaining all the necessary information for the pre-processing phase of the local model. The second step is devoted to the static analysis of the local refined FE model based on the CUF which makes use of BCs at the interface between global and local models. The presented results of the proposed global/local methodology provides good confidence for further work in this direction. Moreover, the use of high-order theories in the local model not only allows dealing with complex 3D stress/strain fields that cannot be detected by using standard FE formulations, but also provides a considerable reduction of the computational costs.

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