Caractérisation multi-échelle des propriétés viscoélastiques des matériaux composites à l'aide d'essais harmoniques

Multi-scale characterisation of viscoelastic properties of composite materials by an harmonic test

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In this paper, the problem of characterising the viscoelastic behaviour of a composite material at each pertinent scale is addressed. To this purpose, a dedicated identification strategy (MSIS) exploiting the information restrained in the macroscopic dynamic response [1, 2] of the composite specimen is developed. The aim of the MSIS [3] is the identification of the viscoelastic behaviour of the composite at both mesoscopic (lamina-level) and microscopic (constitutive phases level) scales. The final goal of the MSIS can be reached by solving an inverse problem (Eq. 2) in which, the identification of the viscoelastic microscopic properties of the constitutive (collected in the vector of design variables $\mathbf{x}$) is obtained by minimising the distance between the numerical $\left(f_r, H_{r,q}\right)$ and the reference $\left(f_r^{\text{ref}}, H_{r,q}^{\text{ref}}\right)$ harmonic macroscopic responses of the specimen (Eq. 1).

$$\Phi(\mathbf{x}) = \frac{1}{N_p \cdot N_s} \sum_{q=1}^{N_p} \sum_{r=1}^{N_s} 2 \cdot \left(\frac{f_r - f_r^{\text{ref}}}{f_r^{\text{ref}}} \right)^2 + \left(\frac{\text{Re}\left(H_{r,q}(\mathbf{x}) - H_{r,q}^{\text{ref}}\right)}{\text{Re}\left(H_{r,q}^{\text{ref}}\right)} \right)^2 + \left(\frac{\Im\left(H_{r,q}(\mathbf{x}) - H_{r,q}^{\text{ref}}\right)}{\Im\left(H_{r,q}^{\text{ref}}\right)} \right)^2. \quad \text{(Eq. 1)}$$

The identification problem can be stated in the form of a classical constrained non-linear programming problem as follows:

$$\begin{cases} \min_{\mathbf{x}} \Phi(\mathbf{x}), \\ \mathbf{g}(\mathbf{x}) \leq 0. \end{cases} \quad \text{(Eq. 2)}$$

The vector of optimisation constraints $\mathbf{g}$ takes into account for different requirements, i.e.

- on the damped natural frequencies of the structure, obtained by solving the non-linear complex eigenvalues problem

$$\left[\left[K_{sys}(\Omega)\right] - \Omega^2 \cdot \left[M_{sys}\right]\right] \left\{U_{sys}\right\} = \{0\}, \quad \text{(Eq. 3)}$$

- existence conditions in order to ensure the positive definitness of the stiffness tensor of each constitutive phase (i.e . fibre and matrix) [3].

It is noteworthy that the damped natural frequencies of (Eq. 3) are the result of a non-linear eigenvalue problem. Indeed, the stiffness matrix of the structure depends upon the frequency, because of the viscoelastic behaviour of the matrix. To solve such a problem a suitable solver based on the Arnoldi's method [4] has been implemented into the finite element code used for the modal analysis. The MSIS relies, on the one hand, on a general homogenisation procedure based on the strain energy of periodic media generalised to the case of viscoelastic materials. On the other hand, the identification process makes use of a general hybrid optimisation algorithm [5] to solve the inverse problem (Fig. 1). The effectiveness of the MSIS is proven through a suitable benchmark.

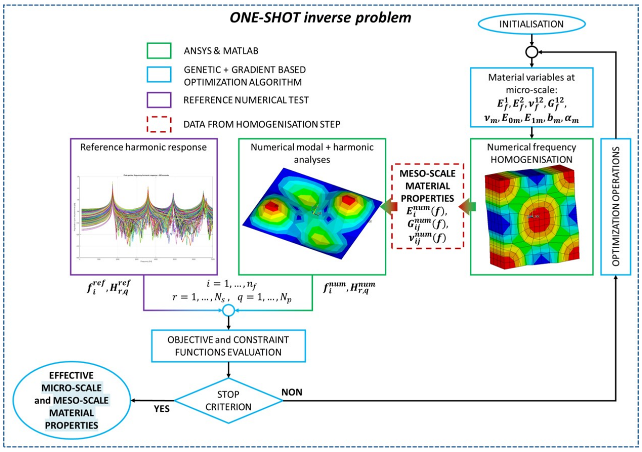

Figure 1.: Optimisation scheme for solution of the inverse problem.

Acknowledgements

This research work has been carried out within the project FULLCOMP (FULLy analysis, design, manufacturing, and health monitoring of COMPosite structures), funded by the European Union Horizon 2020 Research and Innovation program under the Marie Skłodowska-Curie grant agreement No. 642121.

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